Fragment formation in proton induced reactions within a BUU transport model

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The formation of fragments in proton-induced reactions at low relativistic energies within a combination of a covariant dynamical transport model and a statistical approach is investigated. In particular, we discuss in detail the applicability and limitations of such a hybrid model by comparing data on fragmentation at low relativistic SIS/GSI-energies.

Key words: BUU transport equation, statistical multifragmentation model, relativistic proton-nucleus collisions.
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1 Introduction

One of the major aspects in investigating proton induced reactions is to better understand the phenomenon of fragmentation of a nucleus in a hot fireball-like state. Proton induced reactions are the simplest possible way to study such phenomena. Also, they have been found to be important for other investigations, e.g., for the production of radioactive beams [1] or to interpret the origin of cosmic rays and radionuclides in nuclear astrophysics [2]. More recently they have gained again experimental [3] interest. It is therefore a challenge to study this field of research in detail, in particular in relation to the future investigations at the new experimental facility FAIR at GSI.

Proton induced reactions (and also heavy-ion collisions) are usually modeled by non-equilibrium transport models, see for a review Refs. [4–6]. However, the description of fragment formation is a non-trivial task, since transport models do not account for the evolution of physical phase space fluctuations. The major difficulty here is the implementation of the physical fluctuating part of the collision integral and the reduction of numerical fluctuations using

many test particles per physical nucleon, which however would require a large amount of computing resources. Attempts to resolve this still open problem have been recently started [7].

The standard approach of phenomenological coalescence models for fragment formation has been found to work astonishingly well in heavy ion collisions, as long as one considers only one-body dynamical observables, see [8]. In particular, the coalescence model is usually applied in violent heavy-ion collisions, in which a prompt dynamical explosion of a fireball-like system is expected, with the formation of light clusters through nucleon coalescence. In this dilute matter secondary effects are negligible. However, the dynamical situation in proton-induced reactions is different. Compression-expansion effects are here only moderate and the fragmentation process happens over a long time scale (compared to the short lived explosive dynamics in heavy-ion collisions), which is compatible with a statistical description of the process.

The whole dynamical picture in proton-induced reactions is therefore modeled by a combination of dynamical and statistical models. So far, two types of microscopic approaches have been frequently applied in proton-induced reactions: the intranuclear cascade (INC) model [9] and the quantum molecular dynamics (QMD) prescription [10], in combination with a statistical multifragmentation model (SMM) [11]. The SMM model is based on the assumption of an equilibrated source and treats its decay statistically. It includes also sequential evaporation and fission.

In this letter we study fragment formation in proton-induced reactions at low relativistic energies in the framework of a fully covariant coupled-channel transport equation based on the relativistic mean-field approach of Quantum-hadro-dynamics [12]. As a new feature we consider here for the first time the formation of fragments describing the initial step dynamically by means of a covariant transport model of a Boltzmann-Uehling-Uhlenbeck (BUU) type, followed by a statistical formation process of fragments in terms of the SMM model [11]. We compare the theoretical results with a broad selection of experimental data.

2   Theoretical description of proton-induced reactions

The theoretical description of hadron-nucleus (and also heavy-ion reactions) is based on the semiclassical kinetic theory of statistical mechanics, i.e. the Boltzmann Equation with the Uehling-Uhlenbeck modification of the collision integral [5,6]. The covariant analog of this equation is the Relativistic

Boltzmann-Uehling-Uhlenbeck (RBUU) equation [13,14]

$$\left[p^{*\mu}\partial^x_\mu + \left(p^*_\nu F^{\mu\nu} + M^*\partial^\mu_x M^*\right)\partial^{p^*}_\mu \right] f(x,p^*) = \mathcal{I}_{coll} \quad , \tag{1}$$

where $f(x,p^*)$ is the single particle distribution function for the hadrons. The dynamics of the drift term, i.e. the lhs of eq.(1), is determined by the mean field, which does not explicity depend on momentum. Here the attractive scalar field Σ_s enters via the effective mass $M^* = M - \Sigma_s$ and the repulsive vector field Σ_μ via the kinetic momenta $p^*_\mu = p_\mu - \Sigma_\mu$ and via the field tensor $F^{\mu\nu} = \partial^\mu\Sigma^\nu - \partial^\nu\Sigma^\mu$. The dynamical description according to Eq.(1) involves the propagation of hadrons in the nuclear medium, which are coupled through the common mean-field and 2-body collisions. The exact solution of the set of the coupled transport equations is not possible. Here we use the standard test-particle method for the numerical treatment of the Vlasov part. The collision integral, i.e. the rhs of eq.(1), is modeled by a parallel-ensemble Monte-Carlo algorithm.

The results presented here are based on Eq. (1) in a new version of Ref. [15], as realized in the Giessen-BUU (GiBUU) transport model, presented in [15,16], where also the properties of the relativistic mean-field, cross sections and the collision integral are discussed.

Furthermore, we note that the results presented here do not differ essentially from those performed with non-relativistic prescriptions [17], as will be shown below. This is expected, since the achieved energies of the fragmenting sources are smaller than the rest energy. However, a fully covariant description is advantageous for several reasons. First of all, dynamics and kinematics are described on a common level, since the transport equations are formulated in a covariant manner. Apart from that, the relativistic mean-field accounts by definition for higher order momentum dependent effects which would be missed in a non-relativistic approach. The relativistic formulation is also of advantage for other dynamical situations, e.g. heavy-ion collisions at high relativistic energies by studying the formation of hypernuclei. For these reasons we have used a fully covariant approach, which will be applied to the more complex dynamics of relativistic heavy-ion collisions in the future.

The non-linear Walecka model has been adopted for the relativistic mean-field potential. This model gives reasonable values for the compression modulus ($K = 200$ MeV) and the saturation properties of nuclear matter [16,18]. In this first study of multifragmentation (see below) we use its standard version accounting only for the iso-scalar part of the hadronic EoS (σ, ω classical bosonic fields). The exchange of iso-vector bosons (ρ, δ) is neglected in the mean-field baryon potential. At present, we are interested mainly in understanding the fragmentation process in terms of a dynamical description accepting the inaccuracies which might occur in the yields for exotic nuclei, when isovector
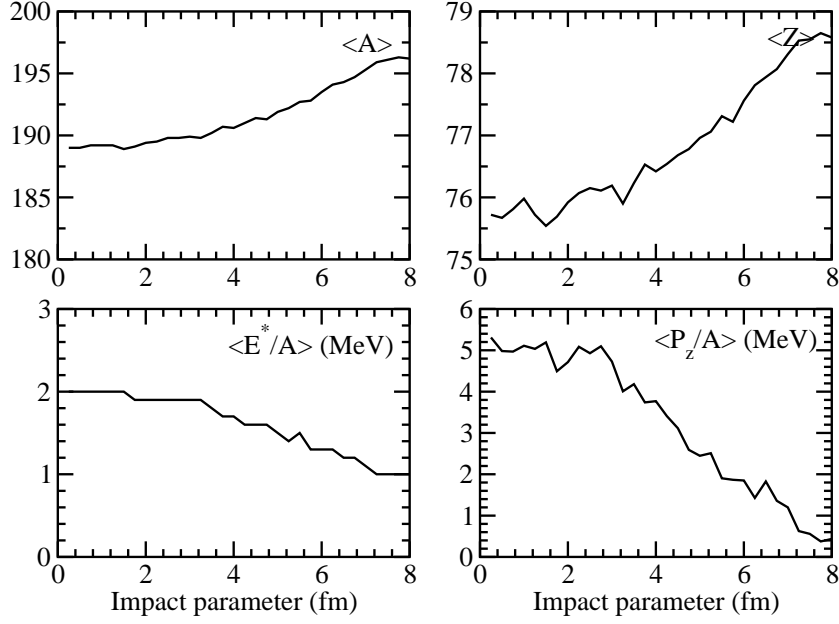
Fig. 1. Impact parameter dependence of the mass (top-left), the charge (top-right panel), excitation energy per nucleon (bottom-left panel) and longitudinal momentum transfer per nucleon (bottom-right panel) of the fragmenting sources produced in p+Au reactions at $E_{lab} = 0.8 GeV$.

interactions are neglected. In order to keep the calculations feasible we have to introduce further approximations. We assume that all baryons, also hadronic resonances and those with finite strangeness, feel the same mean-field. Meson self-energies are not taken into account, except for the Coulomb field.

The transition from the fully dynamical (BUU) to the purely statistical approach (SMM) is not straightforward, and has to be studied carefully. In particular, it involves the time in which one has to switch from the dynamical to the statistical situation. Furthermore, an important feature for proton-induced reactions is the numerical stability of ground state nuclei, in particular, the determination of the ground state binding energies, within the test particle method. In the relativistic non-linear Walecka model the total energy is extracted as the space-integral of the T^{00}-component of the conserved energy-momentum tensor. The phase-space distribution function $f(x, p^*)$ is represented within the test particle *Ansatz*, in which $f(x, p^*)$ is discretized in terms of test particles of a Gaussian shape in coordinate space and a δ-like function in momentum space.

The transition from the dynamical (BUU) to the statistical (SMM) picture is controlled by the onset of local equilibration. We calculate in each time step at the center of the nucleus the spatial diagonal components of the energy momentum tensor $T^{\mu\nu}$ and define the local, e.g. at the center of the source, anisotropy as $Q(x) = \frac{2T^{zz}(x)}{T^{xx}(x)+T^{yy}(x)}$. The onset of local equilibration is defined as that time step, in which the anisotropy ratio approaches unity ($\pm 10\%$). It

turns out that for $p + Au$ reactions at $E_{beam} = 0.8 \, AGeV$ incident energy the system approaches local equilibrium at $t \in [100, \, 120] \, fm/c$, depending on the centrality of the proton-nucleus collision. During the dynamical evolution of a proton-nucleus collision in the spirit of Eq. (1) the nucleus gets excited due to momentum transfer and starts to emit nucleons (pre-equilibrium emission). Assuming that all particles inside the nuclear radius (including its surface) belong to the compound system, it is appropriate to define a *(fragmenting) source* by a density constraint of $\rho_{cut} = \frac{\rho_{sat}}{100}$ ($\rho_{sat} = 0.16 \, fm^{-3}$ being the saturation density). We have checked that the results do not depend on the choice of the density constraint and the resulting difference is less than the statistical fluctuations. Thus, the parameters of the fragmenting source are given by the mass (A), the charge (Z) and the excitation energy at the time of equilibration.

The major parameter entering into the SMM-code is the excitation energy E_{exc} of a source with a given mass (A) and charge (Z) number. Its excitation is obtained by subtracting from the total energy the energy of the ground state, extracted within the same mean-field model as used in the Vlasov equation, for consistency. In a wide time interval from $t \approx 50 \, fm/c$ up to $t_{max} = 150 \, fm/c$, in which one switches from dynamical to the statistical picture, the binding energy per particle of a ground state nucleus remains rather stable with small fluctuations around a mean value in the order of maximal $\pm 1\%$ using 200 test particles per nucleon, which is important in calculating the excitation of the system.

All the theoretical results of the following section have been performed within the *hybrid* approach $GiBUU + SMM$ outlined here. Mass and charge numbers and excitation energy of the fragmenting source have been determined by imposing the density cut after onset of equilibration.

3   Results on $0.8 \, GeV$ $p + Au$ reactions

As a first benchmark we consider the properties of the initial non-equilibrium dynamics and the properties of the fragmenting source, see Fig. 1. During the non-equilibrium dynamics the proton beam collides with nucleons of the target nucleus. The amount of energy transfer and thus of excitation of the residual nucleus with associated particle emission depends on the centrality of the reaction, as shown in Fig. 1. With increasing impact parameter the proton beam experiences less collisions (and also less secondary scattering with associated inelastic processes, e.g. resonance production and absorption) with the particles of the target leading to less energy and momentum transfer. Thus, the pre-equilibrium emission is reduced, as can be clearly seen in Fig. 1. In average, the pre-equilibrium emission takes mainly place in a time interval, in which
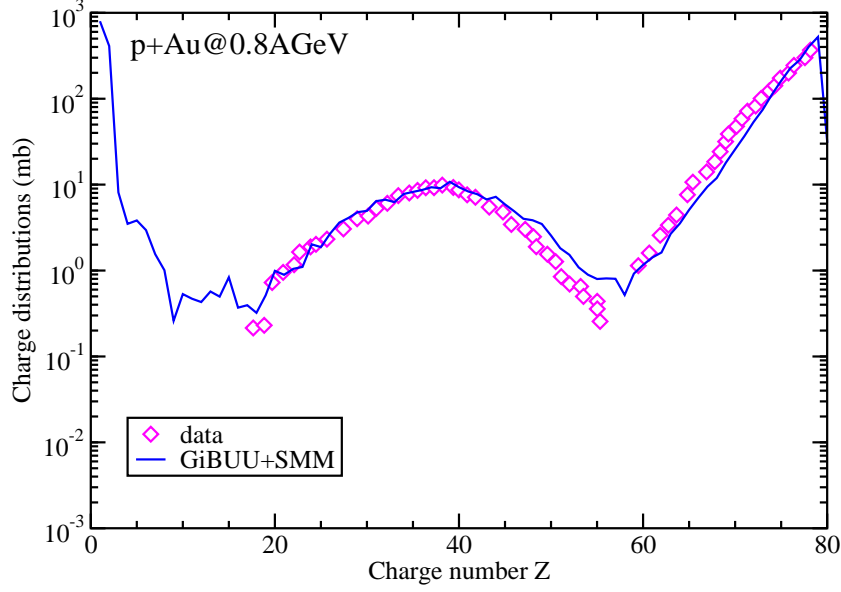
Fig. 2. Charge distributions for $p + Au$ reactions at $E_{beam} = 0.8\ AGeV$ incident energy. Theoretical calculations (solid) are compared with data from [20,21] (open diamonds).

the proton beam penetrates the nucleus. The time interval of pre-equilibrium emission only moderately depends on the impact parameter, in agreement with previous studies, e.g. see Ref. [19]. However, as discussed in the previous section, we stop the dynamical calculation at a time later on, when all resonances have decayed and the residual system has achieved local equilibrium. The average amount of pre-equilibrium emission is $< A_{loss} > \approx 5$, $< Z_{loss} > \approx 3$ in terms of the mass and charge numbers, respectively. During this time interval the excitation of the residual nucleus drops from $< E_{exc}/A > \approx 4.2\ MeV$ to $< E_{exc}/A > \approx (1.5 - 1.7)\ MeV$ due to fast particle emission, before the residual system approaches a stable configuration. We note that our results are in agreement with those of other groups using non-relativistic approaches [17], as expected and discussed above.

The hybrid model discussed in the previous section has been applied to $p + {}^{197}Au$ reactions at low relativistic energies, where a variety of experimental data is available [20,21]. We have used 200 test particles per nucleon for each run and for each impact parameter from $b = 0\ fm$ up to b_{max}.

In Figs. 2, 3 we compare our theoretical results to experimental data [20,21] for $p + Au$ reactions in terms of charge and mass distributions, respectively. The theoretical results are in reasonable agreement both with the *absolute* yields and the shape of the experimental data. Similar results were obtained in previous studies within the the INC model [22].

The fragmentation of an excited system is a complex process involving differ-
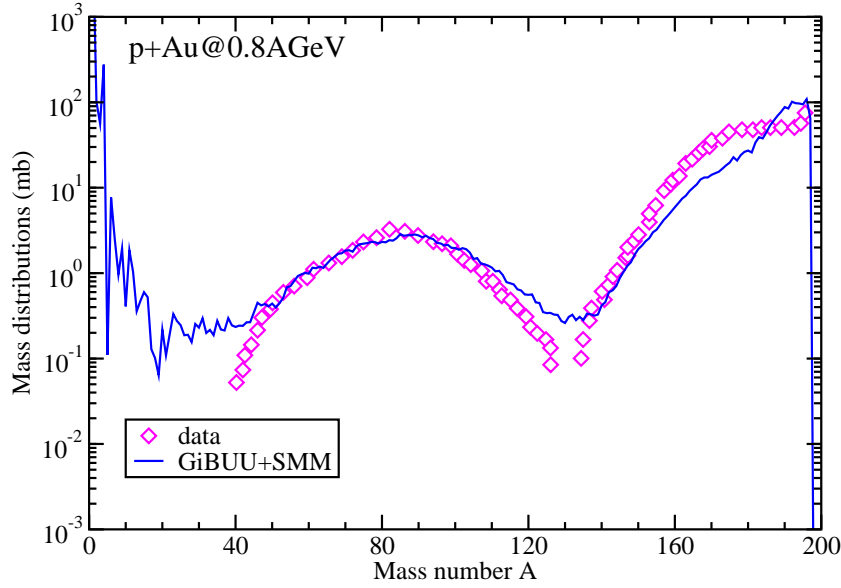
Fig. 3. Same as in Fig. 2 for the mass distributions.

ent mechanisms of dissociation: Sharp peaks at the regions $(A, Z) = (A, Z)_{init}$ and $(A, Z) \approx (A, Z)_{init}/2$, where A_{init}, Z_{init} denote the initial mass and charge numbers, respectively, correspond only to the most peripheral events with very low momentum transfer and thus with low excitation energy. According to the SMM model, heavy nuclei at low excitation energy corresponding to a temperature $T < 2\ MeV$ mainly undergo evaporation and fission producing the sharp peaks at the very high and low mass and charge numbers. With decreasing centrality the excitation energy, respectively temperature, increases. As the excitation energy (or temperature) approaches $T \approx 5\ MeV$ the sharp structure degrades due to the onset of the multifragmentation mechanism, and at higher excitations, $T = 5 - 17\ MeV$, one expects exponentially decreasing yields with decreasing mass/charge number. These different phenomena of dissociation of an excited source finally leads to wide distributions in A, Z as shown in Figs. 2, 3.

The main features of the fragmentation process are apparently reasonably well reproduced by our hybrid approach. The combination of the non-equilibrium dynamics (GiBUU) and the statistical decay of the equilibrated configuration (SMM) obviously accounts for the essential aspects of the reaction. The first stage is important in calculating the excitation of the residual (due to dynamical pre-equilibrium emission) system, and the second one for the statistical fragmentation of the equilibrated configuration.

The situation is quite similar for the mean kinetic energies of produced fragments, displayed in Fig. 4. The average kinetic energies show an almost linear rise from the low-Z and high-Z regions towards the maximum around $Z \approx 20$. Qualitatively, the lightest fragments with the larger slopes are produced in the
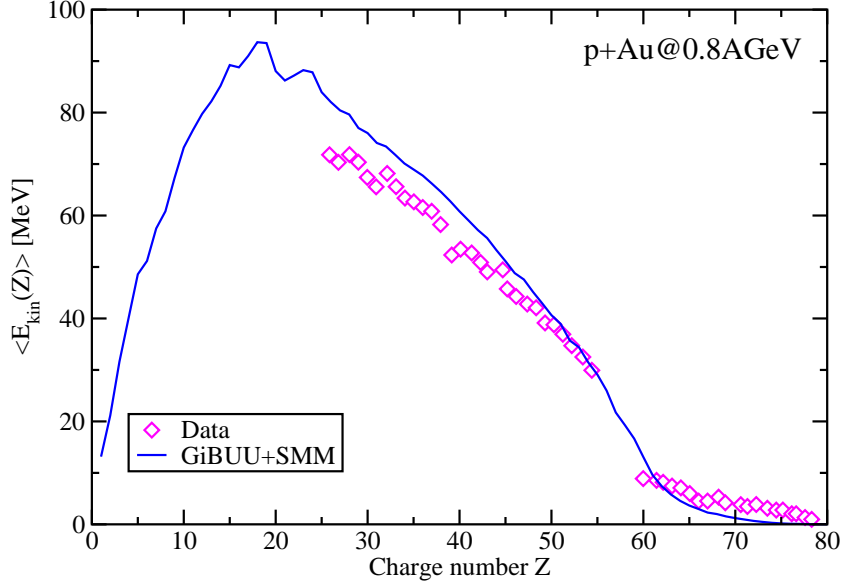
Fig. 4. Same as in Fig. 2 for the average kinetic energies.

most central collisions, corresponding to a large momentum transfer. The low energy tail reflects the most peripheral events with low momentum transfer. It mainly contains the heavy residual products with $Z > 60$.

An exact interpretation of the system size dependence of the average kinetic energies in Fig. 4 is not trivial and would require a detailed discussion of the SMM model, which is not the scope of this work. It is possible, however, to give some quantitative arguments. Quantitatively, for a pure binary fission one would expect naively a maximum of the energy distribution at about half the target charge, $Z = 40$. However, such a picture would neglect the dynamics of the reaction. In order to understand the energy distribution we have to consider the BUU pre-equilibrium dynamics and the fragment formation mechanism. The transport dynamics leads to pre-equilibrium particle emission which results to an initial compound nucleus with smaller Z-values compared to that of the target. Thus the energy naturally becomes smaller. On the other hand, the formation of the fragments is determined by the nuclear binding energies, and their interaction after formation where Coulomb effects play an important role [11]. The binding energy effect favors nuclei around iron ($Z \sim 26$). The Coulomb repulsion leads to long-range correlations among the fragments with a tendency to shift the distribution to slightly smaller Z-values.

The mass and charge distributions of the yields or energies of the produced fragments show only the general trends, which are reproduced well by the theoretical model applied in this work. However, they are not sensitive enough to the details of the reaction. A more stringent test is to study the characteristics of individual nuclides and particles produced in the reaction. Figs. 5 and 6 show theoretical results and experimental data on production yields of
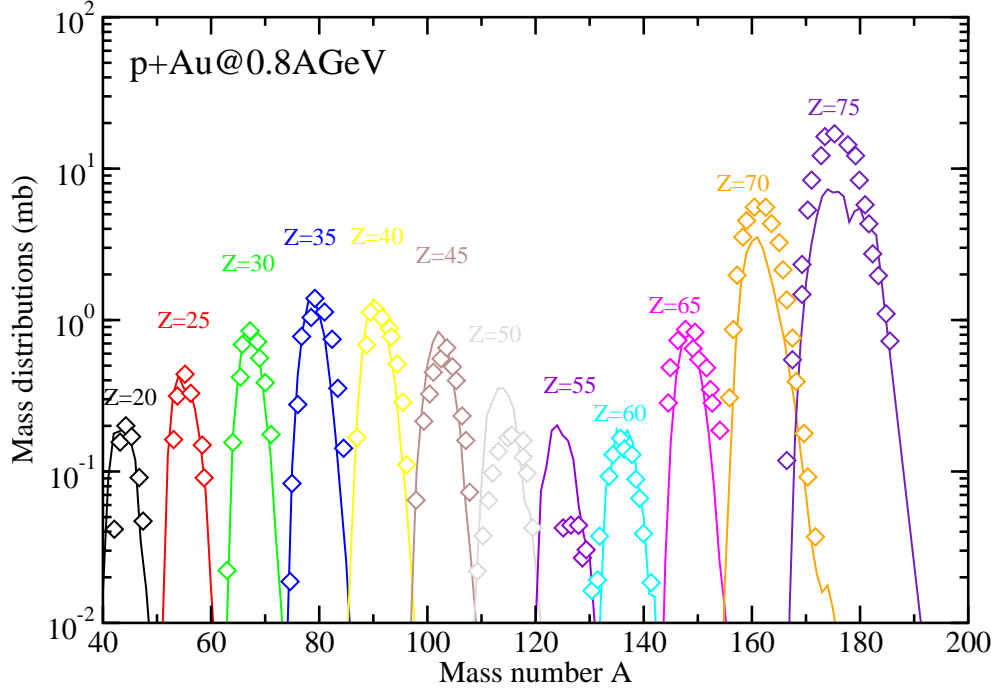
Fig. 5. Same as in Fig. 2 for the mass distributions of different isotopes (indicated by the atomic number Z).

different separate isotopes in the atomic and mass number, respectively. A good overall agreement is achieved, again as a good check of the theoretical hybrid model.

In particular, in Fig. 5 we see that for isotopes produced in the spallation region (not too far from the target mass) and for fission fragments not too far from the maximum yield the comparison between the theory and experiment is only satisfactory. Similar trends are observed in the neutron spectrum of separate isotopes, see again Fig. 6. Interesting is the discrepancy in the proton-rich regions of the isotopic distributions of the heaviest elements, which can be also seen in the global Z- and A-distributions in the corresponding region. This detailed comparison shows the limitations and needed improvements of the hybrid model. The pre-equilibrium dynamics seems to lead to a more excited configuration with respect to the experiment, since all the theoretical distributions are moderately smoother in comparison with the experimental data, which is better visible in the detailed isotopic distributions.

In general, it turns out that the hybrid model gives a quite satisfactory description of fragmentation data, which is a non-trivial task in transport dynamical approaches. We note again, that the non-equilibrium dynamics has been treated in a microscopic way using the relativistic coupled-channel transport approach, which is an important step in extracting the properties of the
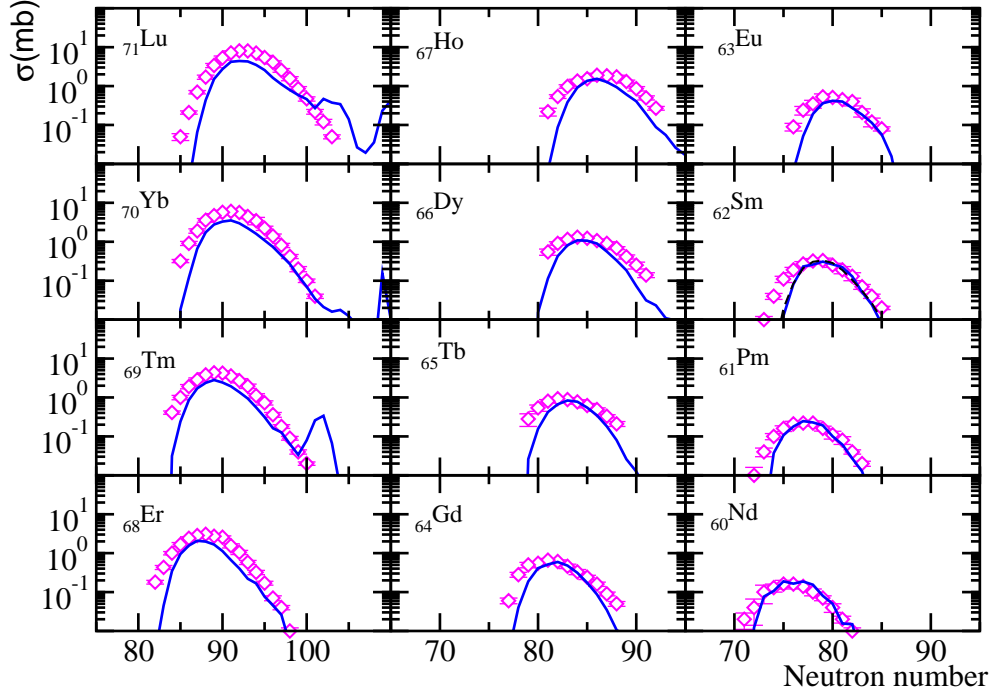
Fig. 6. Same as in Fig. 2 for the neutron distributions of different isotopes (indicated by the atomic number Z). The meaning of the curves is the same as in Fig. 2, expect that the experimental data are taken from [20,21].

fireball-like configuration, before applying its statistical decay in the spirit of the SMM model.

4   Conclusions and outlook

We have investigated the fragmentation mechanism within a hybrid approach consisting of a dynamical transport model of a BUU type and a statistical one in the spirit of the Statistical Multifragmentation Model (SMM), and applied it to low energy proton-induced reactions by means of fragment formation.

The main contribution was to show the reliability and possible limitations of the dynamical transport model for the description of multifragmentation within additional statistical approaches. In particular, it turned out that the hybrid model describes a wide selection of experimental data reasonably well.

As a future project, a consistent description of the initial ground state might be achieved within a semi-classical density functional theory by determining the energy density functional consistently with the density profile of a nucleus implying also the inclusion of surface and isovector contributions to the energy

density and thus to the dynamical evolution. This work is currently in progress.

It is worthwhile to note, that the GiBUU transport approach contains the production and propagation of baryons and mesons with strangeness in appropriate relativistic mean fields. Also the SMM code has been extended to statistical decay of fragments with finite strangeness content (*hypernuclei*) [23]. Therefore, motivated by the results of this work it is straightforward to continue this field of research investigating hypernucleus production from highly energetic nucleus-nucleus collisions. This part of study is still in progress.

In summary, we conclude that this work provides an appropriate theoretical basis for investigations on fragmentation with a new perspective for hypernuclear physics.

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